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Minimum Total-Squared-Correlation Quaternary Signature Sets: New Bounds and Optimal Designs

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***Abstract*—We derive new bounds on the total squared correlation (TSC) of quaternary (quadriphase) signature/sequence sets for all lengths L and set sizes K. Then, for all K, L, we design minimum-TSC optimal sets that meet the new bounds with equality. Direct numerical comparison with the TSC value of the recently obtained optimal binary sets shows under what K, L realizations gains are materialized by moving from the binary to the quaternary code-division multiplexing alphabet. On the other hand, comparison with the Welch TSC value for real/complex-field sets shows that, arguably, not much is to be gained by raising the alphabet size above four for any K, L. The sum-capacity (as well as the maximum squared correlation and total asymptotic efficiency) of minimum TSC quaternary sets is also evaluated in closed-form and contrasted against the sum capacity of minimum-TSC optimal binary and real/complex sets.**

***Index Terms*—Code-division multiplexing, multiuser communications, quadriphase symbols, quaternary alphabet, sequences, sum capacity, Welch bound.**

I. INTRODUCTION

We consider the problem of designing sets of code sequences (signatures) for code-division multiplexing applications from the quaternary alphabet $\{\pm 1, \pm j\}$, $j \triangleq \sqrt{-1}$. A fundamental measure of the cross-correlation properties of a signature set (and subsequently overall code-division system performance) is the total squared correlation (TSC). If $\mathcal{S} \triangleq \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K\}$, $\mathbf{s}_k \in \mathbb{C}^L$, $\|\mathbf{s}_k\| = 1$, $k = 1, 2, \ldots, K$, is a set of K normalized (complex, in general) signatures of length (processing gain) L, then the TSC of $\mathcal{S}$ is the sum of the squared magnitudes of all inner products between signatures, $\mathrm{TSC}(\mathcal{S}) \triangleq \sum_{m=1}^{K}\sum_{n=1}^{K} \left|\mathbf{s}_m^H \mathbf{s}_n\right|^2$ where "H" denotes the conjugate transpose operation. For real/complex-valued signature sets ($\mathcal{S} \in \mathbb{C}^{L\times K}$or $\mathcal{S} \in \mathbb{R}^{L\times K}$), TSC is bounded from below by [1]-[3]

$$\mathrm{TSC}(\mathcal{S}) \geq \frac{KM}{L} \tag{1}$$

where $M = \max\{K, L\}$. The bound in (1) is called the "Welch bound" and the signature sets that satisfy (1) with equality are called Welch-bound-equality (WBE) sets. While for real/complex-valued signature sets the Welch bound is always achievable [4]-[12], this is not the case in general for finite-alphabet signatures. Recently, new bounds were derived for the TSC of binary (alphabet $\{\pm 1\}$) signature sets for all lengths L and set sizes K together with optimal set designs for (almost) all K and L [13]-[15]. The sum capacity, total asymptotic efficiency, and maximum squared correlation of minimum-TSC optimal sets were found in [16]-[17]. Minimum-TSC and other digital sequence sets are studied and utilized in [18]-[21].

The gap in TSC between minimum-TSC binary signature sets and Welch-bound-equality real/complex sets can be reduced if we utilize alphabets of larger size. Certainly there is a tradeoff between system performance and transceiver complexity that is associated with our selection of the alphabet size. The quaternary (or quadriphase or 4-phase) alphabet appears as a good candidate since system complexity increase, which is attributed primarily to the addition of a quadrature signal/carrier into the system, may be negligible. Most code-division multiplexing (CDM) systems already employ quadrature signaling, thus utilizing quaternary signature sets would not incur significant additional cost over binary signature sets.

In this paper, we consider a quaternary alphabet and investigate under what K, L realizations gains can be materialized by moving from the binary to the quaternary code-division multiplexing alphabet. In particular, we first derive new bounds on the TSC of any quaternary signature matrix $\mathcal{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K] \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^{L\times K}$ for all possible K and L values. Then, via quaternary Hadamard matrix transformations, we design minimum-TSC optimal quaternary signature sets that achieve the new bounds. Finally, we derive analytic expressions for the maximum squared correlation (MSC), total asymptotic efficiency (TAE), and sum capacity C_{sum} of the minimum-TSC quaternary sets. In particular, we show that minimum-TSC quaternary sets exhibit the following properties: (*i*) if $K \leq L$, $\mathrm{MSC}(\mathcal{S})$ is also minimum; (*ii*) if $K \leq L$, $\mathrm{TAE}(\mathcal{S})$ is single-valued when $L \equiv 0(\mathrm{mod}\, 2)$ and multi-valued when $L \equiv 1(\mathrm{mod} 2)$; (*iii*) $C_{\mathrm{sum}}(\mathcal{S})$ is single-valued when $\max\{L, K\} \equiv 0(\mathrm{mod}\, 2)$ and multi-valued when $\max\{L, K\} \equiv 1(\mathrm{mod}\, 2)$. We derive the exact value of MSC, TAE, and C_{sum} when these metrics are single-valued. When TAE and/or C_{sum} are multi-valued, we establish lower and upper bounds and prove their tightness; the exact

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value of $C_{\rm sum}$ and/or TAE depends on the particular design of the minimum-TSC signature set. A direct conclusion from this study is that minimum-TSC optimal quaternary sets are not necessarily $C_{\rm sum}$ and/or TAE-optimal, which is also the case for binary antipodal signature sets [16] (we recall that all three metrics are equivalent for real/complex-valued sets [2], [7], [19], [22], [24]). We show, however, that a proposed design of minimum-TSC quaternary signature sets can also minimize MSC, maximize TAE, and maximize $C_{\rm sum}$, simultaneously.

The rest of the paper is organized as follows. In Section II we present the new bounds and optimal designs. In Sections III, IV, and V we evaluate the maximum squared correlation, total asymptotic efficiency, and sum capacity, respectively, of minimum-TSC quaternary sets. A few conclusions are drawn in Section VI.

II. New Bounds on the TSC of Quaternary Signature Sets and Optimal Designs

We recall that the Karystinos-Pados bounds on the TSC of a binary signature set can be given compactly by the following expression [13]-[15]

$$\mathrm{TSC}(\mathcal{S}_B) \geq \frac{KM}{L} + \begin{cases} 0, & M \equiv 0 (\mathrm{mod}\ 4) \\ \frac{m(m-1)}{L^2}, & M \equiv 1 (\mathrm{mod}\ 2) \\ \frac{4}{L^2}[\lfloor \frac{m}{2} \rfloor^2 + \lceil \frac{m}{2} \rceil^2 - m], & M \equiv 2 (\mathrm{mod}\ 4) \end{cases} \tag{2}$$

where K is the number of signatures, L is the signature length, $M = \max\{K, L\}$, and $m = \min\{K, L\}$. The subscript "B" in $\mathcal{S}_B$ identifies a binary signature set $\mathcal{S}_B \in \frac{1}{\sqrt{L}}\{\pm 1\}^{L\times K}$, and $\lfloor x \rfloor$, $\lceil x \rceil$ stand for the closest to x integer that is less than or equal to x and greater than or equal to x, respectively.

In this paper, we consider quaternary signature sets $\mathcal{S}_Q \triangleq [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K] \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^{L\times K}$, $j \triangleq \sqrt{-1}$, where the subscript "Q" in $\mathcal{S}_Q$ stands for quaternary. Since binary signature sets are special cases of quaternary signature sets ($\{\pm 1\} \subset \{\pm 1, \pm j\}$), for any K and L any achievable lower bound on the TSC of $\mathcal{S}_Q$ lies between the Welch bound and the Karystinos-Pados bound. Thus, using (2), whenever $M \equiv 0 (\mathrm{mod}\ 4)$ the bound on TSC of any set $\mathcal{S}_Q$ is

$$\mathrm{TSC}(\mathcal{S}_Q) \geq \begin{cases} K, & K \leq L \text{ and } L \equiv 0 (\mathrm{mod}\ 4), \\ K^2/L, & K > L \text{ and } K \equiv 0 (\mathrm{mod}\ 4). \end{cases} \tag{3}$$

In the rest of this section, we derive new bounds on the TSC of quaternary signature sets for all possible combinations of the values of K and L when $M \equiv 1 (\mathrm{mod} 2)$ and $M \equiv 2 (\mathrm{mod}\, 4)$. Then, via quaternary Hadamard matrix transformations, we design optimal quaternary signature sets that achieve the new bounds.

A. Underloaded system ($K \leq L$)

Theorem 1 below provides new lower bounds on the TSC of any signature set $\mathcal{S}_Q$ when $K \leq L$ (underloaded systems) and $\max\{K, L\} = L$ is not a multiple of 4.

Theorem 1: Let $\mathcal{S}_Q$ be an arbitrary quaternary signature set $\mathcal{S}_Q \triangleq [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K] \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^{L\times K}$, $j = \sqrt{-1}$, $K \leq L$. Then,

$$\mathrm{TSC}(\mathcal{S}_Q) \geq \begin{cases} K + \frac{K(K-1)}{L^2}, & L \equiv 1 (\mathrm{mod}\ 2) \\ K, & L \equiv 0 (\mathrm{mod}\ 2). \end{cases} \tag{4}$$

Proof: The TSC of $\mathcal{S}_Q$ can be expressed as

$$\mathrm{TSC}(\mathcal{S}_Q) = K + \sum_{m=1}^{K} \sum_{\substack{n=1,\\ m\neq n}}^{K} \left| \mathbf{s}_m^H \mathbf{s}_n \right|^2 \tag{5}$$

where the double-summation term is the TSC between *different* signatures in $\mathcal{S}_Q$ (if this term has zero value, i.e. all pairs of signatures have zero cross-correlation, the lower bound of $\mathrm{TSC}(\mathcal{S}_Q)$ reduces to the Welch bound). To obtain a lower bound on the double-summation term in (5), we consider the set $\mathcal{C}$ of all non-ordered pairs of signatures $\{\mathbf{s}_m, \mathbf{s}_n\}$, $m \neq n$, with non-zero cross correlation, i.e. $\mathcal{C}(\{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K\}) \triangleq \{\{\mathbf{s}_m, \mathbf{s}_n\}$ such that $m \neq n$ and $\mathbf{s}_m^H \mathbf{s}_n \neq 0$, $m = 1, 2, \ldots, K$, $n = 1, 2, \ldots, K\}$. Then,

$$\mathrm{TSC}(\mathcal{S}_Q) \geq K + 2\left|\mathcal{C}(\mathcal{S}_Q)\right| |A|^2 \tag{6}$$

where A is a lower bound on the cross-correlation of any two signatures in $\mathcal{S}_Q$ (i.e., $|\mathbf{s}_m^H \mathbf{s}_n| \geq A \; \forall \; \mathbf{s}_m, \mathbf{s}_n \in \mathcal{S}_Q$) and $|\mathcal{C}(\cdot)|$ denotes the cardinality of the set $\mathcal{C}(\cdot)$. Since the quaternary signature alphabet $\frac{1}{\sqrt{L}}\{\pm 1, \pm j\}$ is closed under multiplication and conjugation (denoted by "*"), signature cross-correlations can be expressed as

$$\mathbf{s}_m^H \mathbf{s}_n \triangleq \sum_{i=1}^{L} s_{mi}^* s_{ni} = a\left(\frac{+1}{L}\right) + b\left(\frac{-1}{L}\right) + c\left(\frac{+j}{L}\right) + d\left(\frac{-j}{L}\right) \tag{7}$$

for some integers a, b, c, d such that $a + b + c + d = L$. Then, $\left|\mathbf{s}_m^H \mathbf{s}_n\right| = \frac{1}{L}\sqrt{(a-b)^2 + (c-d)^2}$. If $L \equiv 1 (\mathrm{mod}\ 2)$, $a + b + c + d \equiv 1 (\mathrm{mod}\ 2)$ which implies that $(a-b)^2 + (c-d)^2 \equiv 1 (\mathrm{mod}\ 2)$ and since $(a-b)^2 + (c-d)^2 \geqslant 0$, we have $(a-b)^2 + (c-d)^2 \geqslant 1$. Thus, $\left|\mathbf{s}_m^H \mathbf{s}_n\right| \geq \frac{1}{L}$ for any $\mathbf{s}_m, \mathbf{s}_n \in \mathcal{S}_Q$. We conclude that if $L \equiv 1 (\mathrm{mod}\ 2)$ then the cross-correlation value between any two signatures in $\mathcal{S}_Q$ is non-zero ($|A| = \frac{1}{L}$), therefore $|\mathcal{C}(\mathcal{S}_Q)| = \binom{K}{2} = K(K-1)/2$. On the other hand, if $L \equiv 0 (\mathrm{mod}\ 2)$ there may be signature pairs in $\mathcal{S}_Q$ that exhibit zero cross-correlation. ■

The new bounds on the TSC of quaternary signature sets for underloaded systems ($K \leq L$) are summarized in Table I. Table I can also be viewed as proof that when the signature length is not even, no orthogonal quaternary signature set exists.

B. Overloaded system ($K > L$)

Let $\mathbf{d}_l = [\mathbf{s}_1(l), \mathbf{s}_2(l), \cdots, \mathbf{s}_K(l)]^T \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^K$ denote the transpose of the lth row, $l = 1, 2, \ldots, L$, of the signature matrix $\mathcal{S}_Q$. Due to the "row-column equivalence" [18], $\mathrm{TSC}(\mathcal{S}_Q) = \sum_{m=1}^{K}\sum_{n=1}^{K} \left|\mathbf{s}_m^H \mathbf{s}_n\right|^2 = \sum_{l=1}^{L}\sum_{r=1}^{L} \left|\mathbf{d}_l^H \mathbf{d}_r\right|^2$. Therefore, we can proceed with the calculation of $\mathrm{TSC}(\mathcal{S}_Q)$ as follows

$$\begin{aligned} \mathrm{TSC}(\mathcal{S}_Q) &= \sum_{l=1}^{L} \left|\mathbf{d}_l^H \mathbf{d}_l\right|^2 + \sum_{l=1}^{L} \sum_{r=1, r\neq l}^{L} \left|\mathbf{d}_l^H \mathbf{d}_r\right|^2 \\ &= \frac{K^2}{L} + \sum_{l=1}^{L} \sum_{r=1, r\neq l}^{L} \left|\mathbf{d}_l^H \mathbf{d}_r\right|^2 \\ &= \frac{K^2}{L} + 2\left|C\left(\{\mathbf{d}_1, \mathbf{d}_2, \cdots, \mathbf{d}_L\}\right)\right| \left|\mathbf{d}_l^H \mathbf{d}_r\right|^2 . \end{aligned} \tag{8}$$

TABLE I
UNDERLOADED QUATERNARY SEQUENCE SETS ($K \leq L$)

Length	Number of Sequences	Lower Bound on TSC
$L \equiv 0 (\bmod 2)$	Any K	K
$L \equiv 1 (\bmod 2)$	Any K	$K + \frac{K(K-1)}{L^2}$

TABLE II
OVERLOADED QUATERNARY SEQUENCE SETS ($K \geq L$)

Number of Sequences	Length	Lower Bound on TSC
$K \equiv 0 (\bmod 2)$	Any L	$\frac{K^2}{L}$
$K \equiv 1 (\bmod 2)$	Any L	$\frac{K^2}{L} + \frac{L-1}{L}$

By Theorem 1, we obtain that

$$\mathrm{TSC}(\mathcal{S}_Q) \geq \begin{cases} \frac{K^2}{L}, & K \equiv 0 (\bmod 2) \\ \frac{K^2}{L} + \frac{L-1}{L}, & K \equiv 1 (\bmod 2). \end{cases} \quad (9)$$

The new bounds on the TSC of quaternary signature sets for overloaded ($K > L$) systems are summarized in Table II.

We observe that when $M \triangleq \max\{K, L\}$ is $0 (\bmod 4)$ or $1 (\bmod 2)$ (multiple of four or odd) the TSC lower bounds for quaternary signature sets in Tables I and II are identical to the tight binary Karystinos-Pados (KP) bounds [13], while for $M \equiv 2 (\bmod 4)$, the lower bounds of TSC ($\mathcal{S}_Q$) in Table I and Table II are less than the corresponding KP bounds for binary signature sets. We also recall that $K = L \equiv 1 (\bmod 4)$ is an open standing problem (the only one) [13]-[15] in optimal binary signature set design and KP-bound-equality sets may in fact exist only for values $K = L = 2x(x+1)+1$, $x = 1, 2, \ldots$ (i.e. $K = L = 5, 13, 25, 41, 61, \ldots$). We conclude that we can benefit in TSC by moving from the binary to the quaternary alphabet if $\max\{K, L\}/2$ is an odd integer or $K = L \neq 2x(x+1)+1$, $x = 1, 2, \ldots$. Particularly, when $\max\{K, L\}/2$ is odd, the reduction in TSC is approximately $\frac{4m(m-1)}{L^2}$, $m = \min\{K, L\}$. Moreover, for any K and L, half of the minimum-TSC quaternary sets reach the Welch bound while only a quarter of the minimum-TSC binary sets do. To illustrate the potential reduction in TSC by quaternary designs, in Fig. 1 we consider systems with four different signature lengths $L = 34$, 46, 54, 66 and plot $\mathrm{TSC}(\mathcal{S}) - K$ (i.e. total squared *cross-correlation*) for both binary and quaternary sets as a function of the number of signatures K. To materialize the potential TSC/multiplexing improvements we need designs that meet the corresponding new quaternary bounds in Tables I and II with equality as described in the following subsection.

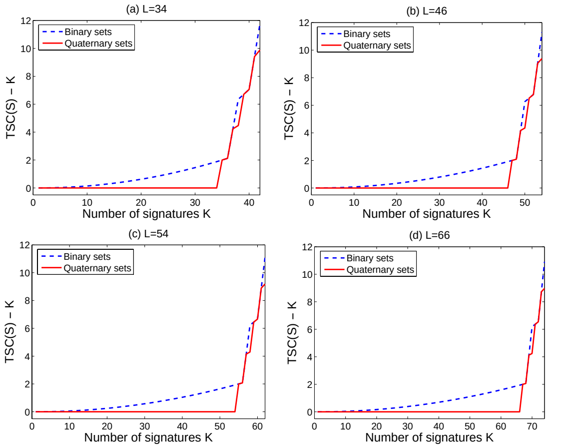

Fig. 1. Total squared *cross-correlation* versus number of signatures K: (a) $L = 34$, (b) $L = 46$, (c) $L = 54$, and (d) $L = 66$.

C. Design of Minimum TSC Quaternary Signature Sets

Our designs of optimal quaternary signature sets are based on transformations of quaternary Hadamard matrices as in [13].

Definition 1: (Quaternary Hadamard matrix)
Let $\mathbf{H}_Q$ be an N-order square matrix over the quaternary alphabet, i.e. $\mathbf{H}_Q \in \{\pm 1, \pm j\}^{N \times N}$, $j \triangleq \sqrt{-1}$, $N > 0$. $\mathbf{H}_Q$ is a quaternary Hadamard matrix if $\mathbf{H}_Q \mathbf{H}_Q^H = N \mathbf{I}_N$ where $\mathbf{I}_N$ is the $N \times N$ identity matrix. ■

Tables I and II indicate that when $M = \max\{K, L\}$ is even, the lower bound on the TSC of quaternary signature sets is equal to the real/complex Welch bound and can be achieved by any set $\mathcal{S}_Q$ that has orthogonal rows when $K \geq L$ or orthogonal columns when $K \leq L$. If M is not even, our lower bounds on the TSC of quaternary signature sets in (4), (9) are strictly larger than the Welch bound which implies that there is no quaternary matrix $\mathcal{S}_Q$ that has orthogonal rows when $K \geq L$ or orthogonal columns when $K \leq L$. In other words, a necessary condition for a quaternary Hadamard matrix to exist is that its size is even[1]; equivalently, if M is not even, then quaternary Hadamard matrices do not exist.

As is the case for binary Hadamard matrices, there is no universal procedure to generate quaternary Hadamard matrices for all even orders. Binary Hadamard matrices can be considered as a special case of quaternary Hadamard matrices. The generation of binary Hadamard matrices of orders that are multiples of four has been well studied. To generate a quaternary Hadamard matrix with order that is a multiple of four, we may multiple by $\pm j$ any column or row of a binary Hadamard matrix of the same order. Unfortunately, it is not easy to generate a quaternary Hadamard matrix of even order N that is not a multiple of four. Below, we propose two alternative methods for this task. First, we suggest a modified version of the analytical procedure of Diţă [25]-[27] that generates $(N-2)^2$ polynomial equations. Our modification incorporates constraints that restrict the phase of the matrix elements to be in the set $\{0, \pi/2, \pi, 3\pi/2\}$. Then, the solutions of the constrained system of $(N-2)^2$ polynomial equations can be used as the entries of a quaternary Hadamard matrix. We note that solving such a constrained system of $(N-2)^2$ polynomial equations is, in general, a computationally complex process and no specific algorithm is available in the literature for this task.

On the other hand, exhaustive search may be thought of as a method to return all quaternary Hadamard matrices of order N. We understand, however, that the complexity of exhaustive search (which is equal to 4^{N^2}) is prohibitively high[2] even for

[1] A necessary condition for a binary Hadamard matrix to exist is that its size is a multiple of four, except for the trivial cases of size one or two.

[2] The complexity may be less if only one quaternary Hadamard matrix needs to be found.

TABLE III
EXAMPLES OF $\mathbf{A}_1$, $\mathbf{A}_2$ CIRCULANT MATRICES

	First row of $\mathbf{A}_1$	First row of $\mathbf{A}_2$
$\frac{N}{2}=3$	$[j\ 1\ 1]$	$[-1\ 1\ 1]$
$\frac{N}{2}=5$	$[1\ j\ -1\ -1\ j]$	$[1\ -1\ j\ j\ -1]$
$\frac{N}{2}=7$	$[1\ 1\ 1\ -1\ -1\ 1\ 1]$	$[1\ j\ -j\ j\ j\ -j\ j]$
$\frac{N}{2}=9$	$[1\ -1\ 1\ 1\ -1\ 1\ -1\ -1\ -1]$	$[1\ 1\ 1\ j\ 1\ 1\ 1\ -1\ -1]$
$\frac{N}{2}=11$	$[1\ -1\ -1\ j\ j\ 1\ 1\ j\ j\ -1\ -1]$	$[j\ 1\ -1\ -j\ j\ -1\ -1\ j\ -j\ -1\ 1]$
$\frac{N}{2}=13$	$[1\ 1\ -1\ j\ j\ -1\ -1\ -1\ -1\ j\ j\ -1\ 1]$	$[j\ 1\ -1\ j\ j\ -1\ -1\ -1\ -1\ j\ j\ -1\ 1]$
$\frac{N}{2}=15$	$[1\ 1\ -1\ j\ -j\ 1\ -1\ -j\ -j\ -1\ 1\ -j\ j\ -1\ 1]$	$[j\ 1\ 1\ j\ j\ -1\ 1\ -j\ -j\ 1\ -1\ j\ j\ 1\ 1]$
$\frac{N}{2}=17$	$[1\ -1\ -1\ j\ -j\ 1\ 1\ j\ j\ j\ j\ 1\ 1\ -j\ j\ -1\ -1]$	$[j\ 1\ -1\ j\ -j\ -1\ -1\ j\ -j\ -j\ j\ -1\ -1\ -j\ j\ -1\ 1]$

moderate values of N. The second method that we suggest herein has significantly less computational complexity (equal to 4^N) than exhaustive search. Our method is based on the following lemma [28], [29].

Lemma 1: If $\mathbf{A}_1$, $\mathbf{A}_2$ are two circulant matrices such that $\mathbf{A}_1, \mathbf{A}_2 \in \{\pm 1, \pm j\}^{\frac{N}{2}\times\frac{N}{2}}$, $\frac{N}{2} \in \mathbb{N}$, and $\mathbf{A}_1$, $\mathbf{A}_2$ satisfy

$$\mathbf{A}_1\mathbf{A}_1^H + \mathbf{A}_2\mathbf{A}_2^H = N\mathbf{I}_{\frac{N}{2}} \tag{10}$$

where $\mathbf{I}_{\frac{N}{2}}$ is an $\frac{N}{2}\times\frac{N}{2}$ identity matrix, then the construction $\mathbf{H}_Q = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_2^H & -\mathbf{A}_1^H \end{bmatrix}$ is an N-order quaternary Hadamard matrix. ■

As an example, if $\mathbf{A}_1 = \begin{bmatrix} j & 1 & 1 \\ 1 & j & 1 \\ 1 & 1 & j \end{bmatrix}$ and $\mathbf{A}_2 = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$, then $\mathbf{H}_Q = \begin{bmatrix} j & 1 & 1 & -1 & 1 & 1 \\ 1 & j & 1 & 1 & -1 & 1 \\ 1 & 1 & j & 1 & 1 & -1 \\ -1 & 1 & 1 & j & -1 & -1 \\ 1 & -1 & 1 & -1 & j & -1 \\ 1 & 1 & -1 & -1 & -1 & j \end{bmatrix}$ is a quaternary Hadamard matrix. There are only $4^{\frac{N}{2}}$ distinct $\frac{N}{2}\times\frac{N}{2}$ circulant matrices over the quaternary alphabet and each of them can be identified by its first row only. If two $\frac{N}{2}\times\frac{N}{2}$ circulant matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ that satisfy (10) exist, they can be found by examining all 4^N possible pairs of circulant matrices. Then, a quaternary Haramard matrix with order N can be generated by $\mathbf{A}_1$ and $\mathbf{A}_2$ as given by Lemma 1. Examples of the first rows of $\mathbf{A}_1$ and $\mathbf{A}_2$-type matrices for different values of N are given in Table III. Additional quaternary Hadamard matrices generated by this method can be found in [30].

In the rest of this section we present a sufficient condition under which the new TSC lower bounds of Tables I and II become tight. Then, we outline a design procedure of quaternary signature sets that achieve the bounds.

Proposition 1: Set $N \triangleq 2\lceil \max\{K,L\}/2 \rceil$ and $P \triangleq 2\lfloor \max\{K,L\}/2 \rfloor$. If there exists a quaternary Hadamard matrix of size N, then for any K and L there exists a quaternary signature matrix $\mathcal{S}_Q = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K] \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^{L\times K}$ that achieves the TSC lower bound in Table I or II. If there exists a quaternary Hadamard matrix of size P, then there exists a quaternary signature matrix $\mathcal{S}_Q = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K] \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^{L\times K}$ with $K \neq L$ that achieves the TSC lower bound in Table I or II. ■

For underloaded systems, $K \leq L$, let $N = 2\lceil L/2 \rceil$ and generate an N-order quaternary Hadamard matrix $\mathbf{H}_Q$. Either $L = N$ or $L = N-1$. If $L = N$, then a quaternary set $\mathcal{S}_Q$ can be formed by selecting and normalizing by $\frac{1}{\sqrt{L}}$ any K columns of $\mathbf{H}_Q$; if $L = N-1$, then we first truncate $\mathbf{H}_Q$ by one row and then form $\mathcal{S}_Q$ by selecting and normalizing by $\frac{1}{\sqrt{L}}$ any K columns from the truncated matrix. For overloaded systems, $K \geq L$, let $N = 2\lceil K/2 \rceil$ and generate an N-order quaternary Hadamard matrix $\mathbf{H}_Q$. Then, $K = N$ or $K = N-1$. If $K = N$, we may choose any L rows of $\mathbf{H}_Q$ and normalize them by $\frac{1}{\sqrt{L}}$; this is our $\mathcal{S}_Q$. If $K = N-1$, we may proceed by truncating $\mathbf{H}_Q$ by one column and then form $\mathcal{S}_Q$ by choosing and normalizing by $\frac{1}{\sqrt{L}}$ any L rows of the truncated matrix.

By Proposition 1, a minimum-TSC quaternary signature set *can also be designed* based on a $P = 2\lfloor \max\{K,L\}/2 \rfloor$ order quaternary Hadamard matrix if it exists. Since $P = N$ when $\max\{K,L\} \equiv 0 (\mathrm{mod} 2)$, we focus on the case $\max\{K,L\} \equiv 1 (\mathrm{mod} 2)$ and $K \neq L$. For underloaded systems, $K < L$, $L \equiv 1 (\mathrm{mod}\ 2)$, and $P = 2\lfloor L/2 \rfloor = L-1$. Generate an $(L-1)$-order quaternary Hadamard matrix $\mathbf{H}_Q$. To form $\mathcal{S}_Q$, we first select any K columns of $\mathbf{H}_Q$, then insert an arbitrary row vector $\mathbf{v}_1^T \in \{\pm 1, \pm j\}^{1\times K}$, and finally normalize all columns by $\frac{1}{\sqrt{L}}$. For overloaded systems, $K > L$, $K \equiv 1 (\mathrm{mod} 2)$, and $P = 2\lfloor K/2 \rfloor = K-1$. Generate a $(K-1)$-order quaternary Hadamard matrix $\mathbf{H}_Q$. To form $\mathcal{S}_Q$ we may proceed by choosing any L rows of $\mathbf{H}_Q$, inserting an arbitrary column vector $\mathbf{v}_2 \in \{\pm 1, \pm j\}^{L\times 1}$, and finally normalizing all rows by $\frac{1}{\sqrt{L}}$.

Fig. 2 summarizes the quaternary signature set design procedure described above in the form of a flow chart subject to the existence of a quaternary Hadamard matrix of order $N = 2\lceil \max\{L,K\}/2 \rceil$ or $P = 2\lfloor \max\{L,K\}/2 \rfloor$. We can show that the TSC of sets $\mathcal{S}_Q$ designed by this procedure is exactly equal to the corresponding new bounds in Tables I or II and thus the produced quaternary signature sets are TSC-optimal.

As an illustrative example, in Fig. 3 we give a TSC-optimal quaternary signature set for an overloaded system with signature length $L = 13$ and $K = 22$ signatures. Another example of optimal design with $L = K = 9$ is shown in Fig. 4. These optimal sets were obtained directly by the design procedure of Fig. 2.

III. Maximum Squared Correlation (MSC) of Minimum-TSC Quaternary Signature Sets

Let $\mathcal{S}_Q = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K]_{L\times K}$ be an underloaded, $K \leq L$, signature matrix with quaternary normalized signatures $\mathbf{s}_k \in$

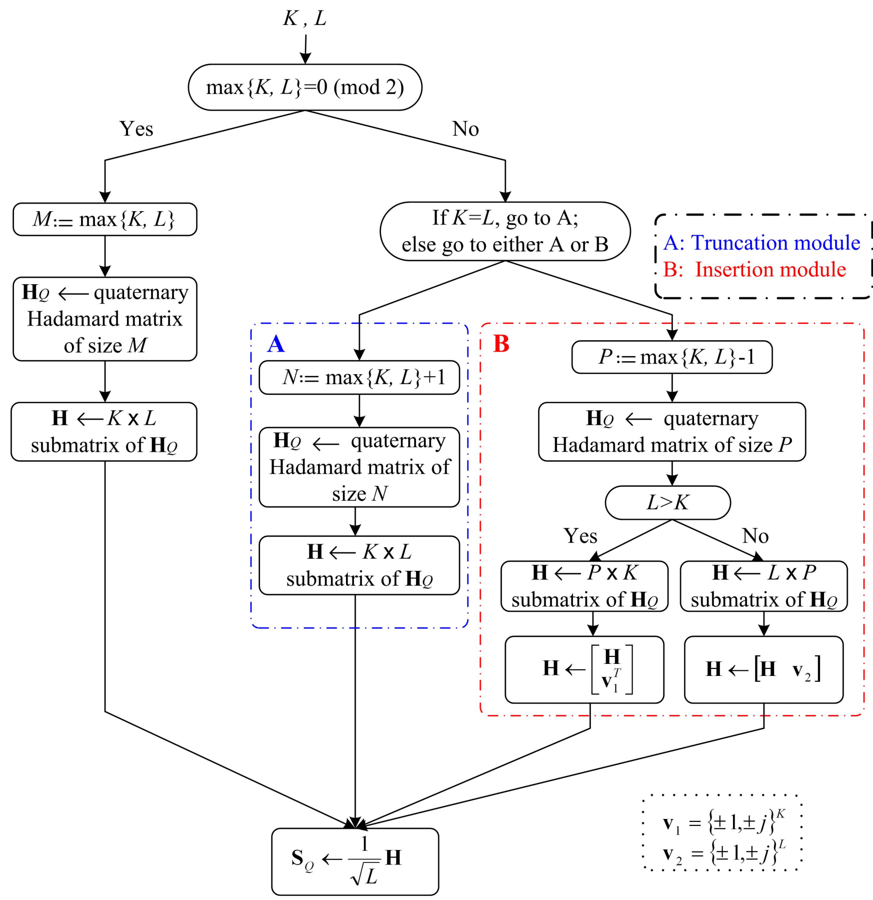

Fig. 2. Optimal quaternary signature set design procedure.

$$\mathcal{S}_Q = \frac{1}{\sqrt{13}} \left[\begin{array}{cccccccccccccccccccccc}
+1 & +1 & +j & -1 & +1 & -j & -j & +1 & -1 & +j & +1 & +j & +1 & -j & +1 & -1 & -j & -j & -1 & +1 & -j & +1 \\
+1 & +1 & +1 & +j & -1 & +1 & -j & -j & +1 & -1 & +j & +1 & +j & +1 & -j & +1 & -1 & -j & -j & -1 & +1 & -j \\
+j & +1 & +1 & +1 & +j & -1 & +1 & -j & -j & +1 & -1 & -j & +1 & +j & +1 & -j & +1 & -1 & -j & -j & -1 & +1 \\
-1 & +j & +1 & +1 & +1 & +j & -1 & +1 & -j & -j & +1 & +1 & -j & +1 & +j & +1 & -j & +1 & -1 & -j & -j & -1 \\
+1 & -1 & +j & +1 & +1 & +1 & +j & -1 & +1 & -j & -j & -1 & +1 & -j & +1 & +j & +1 & -j & +1 & -1 & -j & -j \\
-j & +1 & -1 & +j & +1 & +1 & +1 & +j & -1 & +1 & -j & -j & -1 & +1 & -j & +1 & +j & +1 & -j & +1 & -1 & -j \\
-j & -j & +1 & -1 & +j & +1 & +1 & +1 & +j & -1 & +1 & -j & -j & -1 & +1 & -j & +1 & +j & +1 & -j & +1 & -1 \\
+1 & -j & -j & +1 & -1 & +j & +1 & +1 & +1 & +j & -1 & -1 & -j & -j & -1 & +1 & -j & +1 & +j & +1 & -j & +1 \\
-1 & +1 & -j & -j & +1 & -1 & +j & +1 & +1 & +1 & +j & +1 & -1 & -j & -j & -1 & +1 & -j & +1 & +j & +1 & -j \\
+j & -1 & +1 & -j & -j & +1 & -1 & +j & +1 & +1 & +1 & -j & +1 & -1 & -j & -j & -1 & +1 & -j & +1 & +j & +1 \\
+1 & +j & -1 & +1 & -j & -j & +1 & -1 & +j & +1 & +1 & +1 & -j & +1 & -1 & -j & -j & -1 & +1 & -j & +1 & +j \\
-j & +1 & +j & +1 & -1 & +j & +j & -1 & +1 & +j & +1 & -1 & -1 & +j & +1 & -1 & -j & -j & -1 & +1 & +j & -1 \\
+1 & -j & +1 & +j & +1 & -1 & +j & +j & -1 & +1 & +j & -1 & -1 & -1 & +j & +1 & -1 & -j & -j & -1 & +1 & +j
\end{array}\right]$$

Fig. 3. Optimal quaternary signature set for overloaded multiplexing with signature length $L = 13$ and $K = 22$ signatures.

$\frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^L$, $k = 1, 2, \ldots, K$. We recall that the maximum squared correlation (MSC) of a signature set is the maximum squared magnitude among all inner products between distinct signatures. By the proof of *Theorem 1* in Section II, we can obtain that the maximum squared correlation of $\mathcal{S}_Q$, denoted by $\mathrm{MSC}(\mathcal{S}_Q)$, is lower-bounded as follows:

$$\mathrm{MSC}(\mathcal{S}_Q) = \max_{m \neq n} \left|\mathbf{s}_m^H \mathbf{s}_n\right|^2 \geq \begin{cases} 0, & L \equiv 0 (\mathrm{mod}\, 2) \\ \frac{1}{L^2}, & L \equiv 1 (\mathrm{mod}\, 2). \end{cases} \tag{11}$$

The following two Propositions summarize our findings about the MSC of underloaded minimum-TSC quaternary signature sets. The proof is obtained directly from the material in Section II and is, therefore, omitted.

Proposition 2: Let $\mathcal{S}_Q \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^{L \times K}$, $1 < K \leq L$, be a quaternary signature matrix that achieves the corresponding TSC lower bound in Table I. Then,

(*i*) $\mathrm{MSC}(\mathcal{S}_Q) = 0$, if $L \equiv 0 (\mathrm{mod}\, 2)$;
(*ii*) $\mathrm{MSC}(\mathcal{S}_Q) = \frac{1}{L^2}$, if $L \equiv 1 (\mathrm{mod}\, 2)$. ∎

Proposition 3: An underloaded quaternary signature set achieves the lower bound on TSC in Table I if and only if it achieves the lower bound on MSC in (11). ∎

We conclude that the MSC of minimum-TSC quaternary underloaded sets is less than the MSC of minimum-TSC

$$\mathcal{S}_Q = \frac{1}{3}\begin{bmatrix} +1 & +j & -1 & -1 & +j & +1 & -1 & +j & +j \\ +j & +1 & +j & -1 & -1 & -1 & +1 & -1 & +j \\ -1 & +j & +1 & +j & -1 & +j & -1 & +1 & -1 \\ -1 & -1 & +j & +1 & +j & +j & +j & -1 & +1 \\ +j & -1 & -1 & +j & +1 & -1 & +j & +j & -1 \\ +1 & -1 & -j & -j & -1 & -1 & +j & +1 & +1 \\ -1 & +1 & -1 & -j & -j & +j & -1 & +j & +1 \\ -j & -1 & +1 & -1 & -j & +1 & +j & -1 & +j \\ -j & -j & -1 & +1 & -1 & +1 & +1 & +j & -1 \end{bmatrix}$$

Fig. 4. Optimal quaternary signature set with signature length $L = 9$ and $K = 9$ signatures.

binary set by $\frac{4}{L^2}$ when $L \equiv 2 (\bmod 4)$ and $K > 2$. Most importantly, for all K, L with $K \leq L$, the minimum-TSC quaternary signature sets obtained in Section II are doubly optimal (subject to the existence of a quaternary Hadamard matrix of size $2\lceil L/2 \rceil$). They exhibit minimum both TSC and MSC at the same time. Therefore, when we design quaternary signature sets (Fig. 2) we can focus on minimizing TSC only and can rest assured that MSC will also be minimized. It is interesting to note that the equivalence between TSC and MSC optimization is *not true*, in general, for binary sets[3] [16].

IV. Total Asymptotic Efficiency (TAE) of Minimum-TSC Quaternary Signature Sets

The TAE of a complex-valued signature matrix $\mathcal{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_K]$, $\mathbf{s}_k \in \mathbb{C}^L$, $\|\mathbf{s}_k\| = 1$, $k = 1, 2, \ldots, K$, is equal to the determinant of the signature cross-correlation matrix, $\mathrm{TAE}(\mathcal{S}) \triangleq \left|\mathcal{S}^H \mathcal{S}\right|$ and $0 \leq \mathrm{TAE}(\mathcal{S}) \leq 1$. Since $\mathcal{S}^H\mathcal{S}$ is rank-deficient and $\mathrm{TAE}(\mathcal{S}) = 0$ when $K > L$ (overloaded system), we only consider the underloaded case. $\mathrm{TAE}(\mathcal{S})$ achieves the unit upper bound if $\mathcal{S}$ has orthogonal columns. However, it has been an open question whether tightness is maintained when $\mathcal{S}$ is quaternary, that is $\mathbf{s}_k \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^L$, $k = 1, 2, \ldots, K$. In this section, we obtain closed form expressions for the TAE of minimum-TSC quaternary signature sets for all $K \leq L$. Our developments are based on the proposition that we state below and prove in the Appendix.

Proposition 4: Let $\mathcal{S}_Q \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^{L \times K}$, $K \leq L$, be a quaternary signature matrix that achieves the corresponding TSC lower bound in Table I and $[\mathcal{S}_Q^H \mathcal{S}_Q]_{mn}$ denotes the (m, n)th element of $\mathcal{S}_Q^H \mathcal{S}_Q$, $m = 1, 2, \ldots, K$, $n = 1, 2, \ldots, K$. Then, $\mathcal{S}_Q^H \mathcal{S}_Q$ has following properties:

(*i*) If $L \equiv 0 (\bmod 2)$, $\mathcal{S}_Q^H \mathcal{S}_Q = \mathbf{I}_K$;

(*ii*) if $L \equiv 1 (\bmod 2)$, then $[\mathcal{S}_Q^H \mathcal{S}_Q]_{mm} = 1$ and $[\mathcal{S}_Q^H \mathcal{S}_Q]_{mn} \in \frac{1}{L}\{\pm 1, \pm j\}$, $m \neq n$, $m = 1, 2, \ldots, K$, $n = 1, 2, \ldots, K$;

(*iii*) if $L \equiv 1 (\bmod 2)$ and there exists a quaternary Hadamard matrix $\mathbf{H}_Q$ of size $L + 1$, we can obtain a minimum-TSC signature set which has $[\mathcal{S}_Q^H \mathcal{S}_Q]_{mn} = -\frac{1}{L}$, $m \neq n$, $m = 1, 2, \ldots, K$, $n = 1, 2, \ldots, K$;

(*iv*) if $L \equiv 1 (\bmod 2)$ and there exists a quaternary Hadamard matrix $\mathbf{H}_Q$ of size $L - 1$ and $K \leq L - 1$, we can obtain a minimum-TSC signature set which has $[\mathcal{S}_Q^H \mathcal{S}_Q]_{mn} = \frac{1}{L}$, $m \neq n$, $m = 1, 2, \ldots, K$, $n = 1, 2, \ldots, K$. ■

Based on the above proposition, the TAE of an underloaded minimum-TSC quaternary signature set can be derived and the findings are presented in the form of a proposition given below. The proof is given in the Appendix.

Proposition 5: Let $\mathcal{S}_Q \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^{L \times K}$, $K \leq L$, be a quaternary signature matrix that achieves the corresponding TSC lower bound in Table I. Then,

(*i*) $\mathrm{TAE}(\mathcal{S}_Q) = 1$, if $L \equiv 0 (\bmod 2)$;

(*ii*) $\frac{(L+1)^{K-1}(L-K+1)}{L^K} \leq \mathrm{TAE}(\mathcal{S}_Q) \leq \frac{(L-1)^{K-1}(L+K-1)}{L^K}$, if $L \equiv 1 (\bmod 2)$. The lower bound is tight if there exists a quaternary Hadamard matrix of size $L + 1$ while the upper bound is tight if $K \leq L - 1$ and there exists a quaternary Hadamard matrix of size $L - 1$. ■

We recall that for real/complex-valued sets TAE maximization and TSC minimization are equivalent problems for all K, L with $K \leq L$ [24]. As shown by Proposition 5, however, this property no longer holds true for quaternary signature sets. If $L \equiv 1 (\bmod 2)$ and $K < L$, then there exist minimum-TSC sets that do not have maximum TAE. Indeed, by Proposition 5, minimum-TSC sets created by the truncation module A in the design flow-chart of Fig. 2 are not maximum-TAE, while alternative minimum-TSC designs by the insertion module B are. We also observe that minimum-TSC quaternary sets always exhibit larger TAE values than minimum-TSC binary sets developed in [16] when $L \equiv 2 (\bmod 4)$ while they exhibit larger or equal TAE values when $L \equiv 3 (\bmod 4)$. To illustrate the improvement on TAE of minimum-TSC quaternary sets, in Fig. 5 we plot the TAE[4] of minimum-TSC quaternary and binary sets as a function of K for four different signature length values $L = 31$, 34, 63, and 64. We observe that minimum-TSC quaternary sets exhibit significant TAE improvement relative to their binary counterparts when $L \equiv 2 (\bmod 4)$ and $L \equiv 3 (\bmod 4)$.

V. Sum Capacity of Minimum-TSC Quaternary Signature Sets

The sum capacity C_{sum} of a multiple-access communication channel is the maximum sum of user transmission rates at which reliable decoding at the receiver end is possible [2], [22], [23]. In a synchronous code-division multiple-access system that employs an $L \times K$ complex-valued signature matrix $\mathcal{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K]$, $\mathbf{s}_k \in \mathbb{C}^L$, $\|\mathbf{s}_k\| = 1$, $k = 1, 2, \ldots, K$, for transmissions over a common additive white Gaussian noise (AWGN) channel, the received data vector is of the form $\mathbf{r} = \sum_{k=1}^{K} d_k \mathbf{s}_k + \mathbf{n}$ where $d_k \in \mathbb{C}$, $k = 1, 2, \ldots, K$, is the kth user transmitted symbol (complex in general) and $\mathbf{n}$ is a zero-mean complex Gaussian vector with auto-covariance matrix $N_0 \mathbf{I}_L$. If $E\{|d_k|^2\} = E$, $k = 1, 2, \ldots, K$, it is known [2], [22] that

$$C_{\mathrm{sum}} \triangleq \log_2 \left|\mathbf{I}_L + \gamma \mathcal{S}\mathcal{S}^H\right| = \log_2 \left|\mathbf{I}_K + \gamma \mathcal{S}^H \mathcal{S}\right| \qquad (12)$$

[3]TSC and MSC minimization are equivalent for binary sets for any K, L with $K \leq L$ (subject to the existence of a binary Hadamard matrix of size $4\lfloor \frac{L+2}{4} \rfloor$) except for $L \equiv 2 (\bmod 4)$ or $L = K \equiv 1 (\bmod 4)$.

[4]The design of maximum-TAE minimum-TSC quaternary signature sets is achieved by module B of Fig. 2. In Fig. 5, we only consider the maximum value of TAE when it is multi-valued.

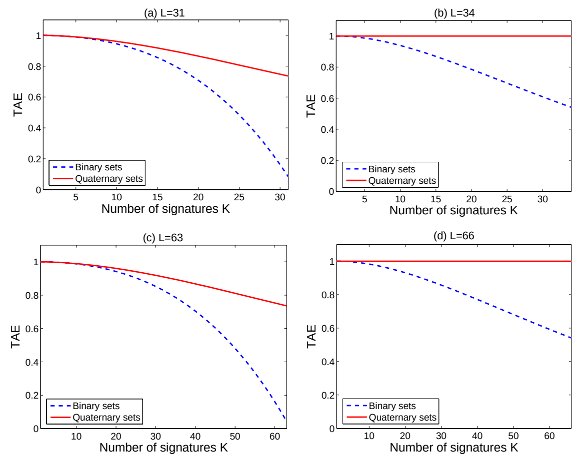

Fig. 5. TAE of binary signature sets and quaternary signature sets versus number of signatures K of length (a) $L = 31$, (b) $L = 34$, (c) $L = 63$, and (d) $L = 66$.

where $\gamma \triangleq \frac{E}{N_0}$ is the received signal-to-noise ratio (SNR) of each user signal and $\mathbf{I}_L$, $\mathbf{I}_K$ are the size-L and size-K identity matrices. It is also well known that the sum capacity is bounded as follows [2], [7], [19]

$$0 \leq C_{\text{sum}}(\mathcal{S}) \leq \begin{cases} K\log_2(1+\gamma), & K \leq L \\ L\log_2(1+\frac{K}{L}\gamma), & K \geq L. \end{cases} \quad (13)$$

While the upper bound in (13) is tight for real/complex-valued signature sets for any K, L, it has been shown in [16] that tightness is *not* always maintained if $\mathcal{S}$ is *binary*. In this section, we consider minimum-TSC quaternary signature sets $\mathcal{S}_Q$ and obtain closed-form expressions for C_{sum} for any K, L. Our developments are presented in the form of a proposition given below. The proof is given in the Appendix.

Proposition 6: Let $\mathcal{S}_Q \in \frac{1}{\sqrt{L}}\{\pm 1, \pm j\}^{L\times K}$ be a quaternary signature matrix that achieves the corresponding TSC lower bound in Table I or Table II. Then,

A) if $K \leq L$ (underloaded system)

(*i*) $C_{\text{sum}}(\mathcal{S}_Q) = K\log_2(1+\gamma)$, if $L \equiv 0(\bmod 2)$;

(*ii*) $(K-1)\log_2(1+\frac{L+1}{L}\gamma) + \log_2(1+\frac{L-K+1}{L}\gamma) \leq C_{\text{sum}}(\mathcal{S}_Q) \leq (K-1)\log_2(1+\frac{L-1}{L}\gamma) + \log_2(1+\frac{L+K-1}{L}\gamma)$, if $L \equiv 1(\bmod 2)$. The lower bound is tight if there exists a quaternary Hadamard matrix of size $L+1$, while the upper bound is tight if $K \leq L-1$ and there exists a quaternary Hadamard matrix of size $L-1$.

B) If $K \geq L$ (overloaded system)

(*i*) $C_{\text{sum}}(\mathcal{S}_Q) = L\log_2(1+\frac{K}{L}\gamma)$, if $K \equiv 0(\bmod 2)$;

(*ii*) $(L-1)\log_2(1+\frac{K+1}{L}\gamma) + \log_2(1+\frac{K-L+1}{L}\gamma) \leq C_{\text{sum}}(\mathcal{S}_Q) \leq (L-1)\log_2(1+\frac{K-1}{L}\gamma) + \log_2(1+\frac{K+L-1}{L}\gamma)$, if $K \equiv 1(\bmod 2)$. The lower bound in (*ii*) is tight if there exists a quaternary Hadamard matrix of size $K+1$ while the upper bound is tight if $L \leq K-1$ and there exists a quaternary Hadamard matrix of size $K-1$. ■

Comparing Proposition 6 with expression (13) for real/complex-valued sets, we see that minimum-TSC quaternary signature sets meet the upper bound in (13) only if $L \equiv 0(\bmod 2)$ for underloaded systems or $K \equiv 0(\bmod 2)$ for overloaded systems. In addition, by Proposition 6, when $L \equiv 1(\bmod 2)$ for underloaded systems or $K \equiv 1(\bmod 2)$ for overloaded systems and $K \neq L$, there exist quaternary minimum-TSC sets that do not exhibit maximum sum capacity. Thus, minimum-TSC and maximum-C_{sum} criteria are not equivalent, in general, for quaternary sets for all K, L. In particular, by Proposition 6, design module B of Fig. 2 produces C_{sum}-optimal minimum-TSC designs, while module A produces minimum-TSC designs that are not C_{sum}-optimal.

Furthermore, comparing Proposition 6 with Proposition 2 in [16] for binary sets, we notice that minimum-TSC quaternary signature sets have higher sum-capacity than minimum-TSC binary signature sets when $L \equiv 2(\bmod 4)$ for underloaded systems and $K \equiv 2(\bmod 4)$ for overloaded systems. More importantly, similar to the TAE metric, C_{sum}-optimal minimum-TSC quaternary signature sets can be produced by module B of Fig. 2, while the design of minimum-TSC binary sets that maximize C_{sum} is an open problem.

To visualize the theoretical developments of Proposition 6 on the sum capacity of quaternary signature sets, we consider the relative sum-capacity-loss expression

$$\Delta(\mathcal{S}) \triangleq 1 - \frac{C_{\text{sum}}(\mathcal{S})}{C^*_{\text{sum}}} \quad (14)$$

where C^*_{sum} is the sum capacity of a real/complex-valued Welch-bound-equality (WBE) signature set of the same size as $\mathcal{S}$. In Fig. 6, we plot the sum-capacity-loss $\Delta(\mathcal{S})$ of minimum-TSC quaternary sets as a function of K for a common received SNR per user $\gamma = 12$ dB and four different signature length values $L = 31$, 32, 33, and 34. Whenever C_{sum} of minimum-TSC quaternary sets is multi-valued, we use the maximum C_{sum} value (module-B produced set). For fair comparison, maximum C_{sum} values are also used for the minimum-TSC binary sets when their corresponding C_{sum} is multi-valued[5] [16]. We observe that minimum-TSC quaternary sets exhibit rather negligible sum-capacity-loss for almost all K, L in comparison with WBE real/complex-valued sets. In addition, the sum-capacity-loss of quaternary minimum-TSC sets is quite less than the sum-capacity loss of binary minimum-TSC sets when K is near L. In Fig. 7, we repeat the same study as in Fig. 6 for $L = 63$, 64, 65, and 66. Similar conclusions can be drawn. It can be argued that sum-capacity-wise it is not worth raising the code-division alphabet size above four for any K, L, since the sum-capacity-loss of minimum-TSC quaternary sets already approaches zero rather closely.

VI. Conclusions

In an effort to gain better understanding of the theoretical intricacies of finite-alphabet code-division multiplexing, we examined the following four signature performance metrics: Total squared correlation (TSC), maximum squared correlation (MSC), total asymptotic efficiency (TAE), and sum capacity (C_{sum}). In this paper, we derived new bounds on the TSC of *quaternary* signature sets for both underloaded and overloaded

[5] The sum-capacity-loss study in [16] used instead the smallest C_{sum} value when multiple values exist among min-TSC binary sets of a given (K, L) size.

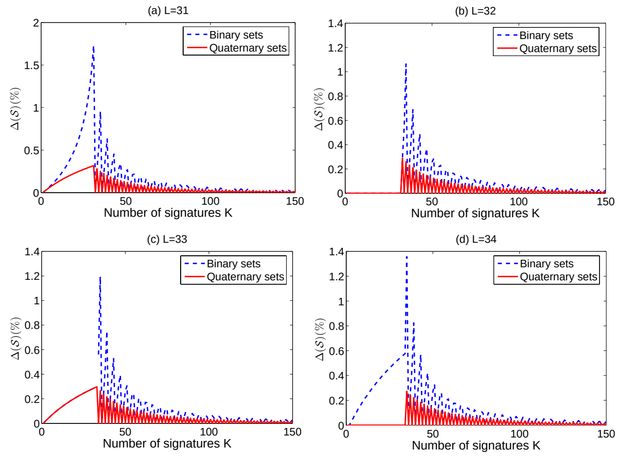

Fig. 6. Sum-capacity loss $\Delta(\mathcal{S})(\%)$ of minimum-TSC binary and quaternary signature sets versus number of signatures K of length (a) $L = 31$, (b) $L = 32$, (c) $L = 33$, and (d) $L = 34$ ($\gamma = 12$ dB).

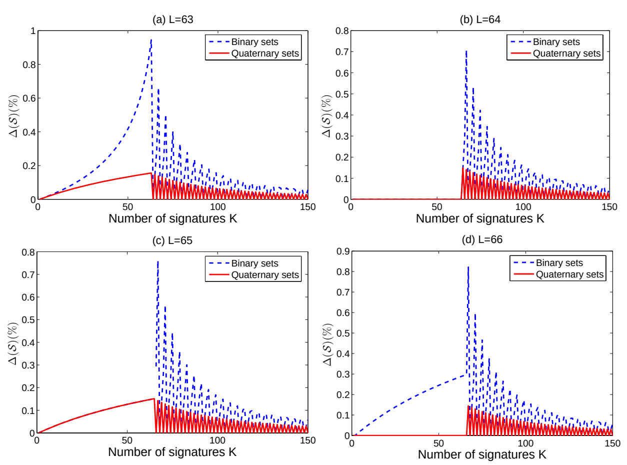

Fig. 7. Sum-capacity loss $\Delta(\mathcal{S})(\%)$ of minimum-TSC binary and quaternary signature sets versus number of signatures K of length (a) $L = 63$, (b) $L = 64$, (c) $L = 65$, and (d) $L = 66$ ($\gamma = 12$ dB).

code-division multiplexing systems (summarized in Tables I and II, respectively). We showed that the new bounds on the TSC of quaternary signature sets are lower than the corresponding binary signature set bounds (same number of signals K and signature length L) for all $\max\{K, L\} \equiv 2(\mathrm{mod}\,4)$ or $K = L \equiv 1(\mathrm{mod}\,4) \neq 2x(x+1)+1$ cases. We then designed minimum-TSC optimal quaternary sets that meet the new bounds for all K, L. Our design procedure depends on the existence of a quaternary Hadamard matrix of size $2\lceil \max\{K, L\}/2 \rceil$ or $2\lfloor \max\{K, L\}/2 \rfloor$.

Utilizing our developments on the TSC of quaternary signature sets, we derived closed-form expressions for the MSC, TAE, and sum capacity that minimum-TSC quaternary signature sets achieve for all K, L with $K \leq L$ and the sum capacity that minimum-TSC quaternary sets achieve for all K, L with $K > L$. We recall that minimum-TSC, minimum-MSC, maximum-TAE, and maximum-sum-capacity are equivalent optimization criteria for real/complex-valued signature sets, i.e. real/complex-valued minimum-TSC signature sets are minimum-MSC and maximum-TAE when the number of signatures K is less than or equal to the signature length L and have maximum sum-capacity for any K, L. Interestingly, for quaternary (and binary [16]) signature sets, there exist K, L values for which different metrics are optimized by different sets. Our studies showed that the sum-capacity loss of the minimum-TSC quaternary signature sets is negligible in comparison with minimum-TSC real/complex-alphabet (Welch-bound-equality) sets and quite smaller than that exhibited by minimum-TSC binary signature sets.

APPENDIX

PROOF OF PROPOSITION 4

The proof of parts (*i*) and (*ii*) can be obtained directly from the proof of *Theorem 1* and is omitted herein. With respect to part (*iii*), we recall that if the rows and columns of a quaternary Hadamard matrix are permuted or any row or column is multiplied by -1 or $\pm j$, the Hadamard orthogonality property is retained. Hence, we can always arrange one row or one column of a quaternary Hadamard matrix to have only $+1$ entries. If there exists a quaternary Hadamard matrix $\mathbf{H}_Q$ of size $L+1$ and $L \equiv 1(\mathrm{mod}\ 2)$, a minimum-TSC signature set can be obtained by taking K columns from $\mathbf{H}_Q$ and removing one row which contains only $+1$ entries. After normalization, the cross-correlation matrix of the created minimum-TSC signature set is

$$\mathcal{S}_Q^H \mathcal{S}_Q = \frac{L+1}{L}\mathbf{I}_K - \frac{1}{L}\mathbf{1}_K \mathbf{1}_K^T \tag{15}$$

where $\mathbf{1}_K$ is the K-dimensional all-one column vector. With respect to part (*iv*), if there exists a quaternary Hadamard matrix $\mathbf{H}_Q$ of size $L-1$ and $K \leq L-1$, a minimum-TSC signature set can be obtained by appending an all-one row $\mathbf{1}_{L-1}^T$ to $\mathbf{H}_Q$ and taking K columns. After normalization, the cross-correlation matrix of the created minimum-TSC signature set is

$$\mathcal{S}_Q^H \mathcal{S}_Q = \frac{L-1}{L}\mathbf{I}_K + \frac{1}{L}\mathbf{1}_K \mathbf{1}_K^T. \tag{16}$$

■

PROOF OF PROPOSITION 5

(*i*) When $L \equiv 0(\mathrm{mod}\ 2)$ and $\mathcal{S}_Q$ achieves the TSC lower bound in Table I, by Proposition 4, part (*i*), we obtain $\mathrm{TAE}(\mathcal{S}_Q) = \left|\mathcal{S}_Q^H \mathcal{S}_Q\right| = |\mathbf{I}| = 1$.

(*ii*) By Proposition 4, $[\mathcal{S}_Q^H \mathcal{S}_Q]_{mm} = 1$ and $\left|[\mathcal{S}_Q^H \mathcal{S}_Q]_{mn}\right| = \frac{1}{L}$, $m \neq n$, $m = 1, 2, \ldots, K$, $n = 1, 2, \ldots, K$. Then, by *Lemma 2* of [16], we obtain that

$$\begin{aligned}(1+\frac{1}{L})^{K-1}(1-(K-1)\frac{1}{L}) &\leq \left|\mathcal{S}_Q^H \mathcal{S}_Q\right| \\ &\leq (1-\frac{1}{L})^{K-1}(1+(K-1)\frac{1}{L}).\end{aligned} \tag{17}$$

Expression (17) leads to the bounds on TAE as they appear in Proposition 5. If there exists a quaternary Hadamard matrix $\mathbf{H}_Q$ of size $L+1$, by (15) we can obtain a minimum-TSC quaternary set which has

$$\begin{aligned}\left|\mathcal{S}_Q^H \mathcal{S}_Q\right| &= \left|\frac{L+1}{L}\mathbf{I}_K - \frac{1}{L}\mathbf{1}_K \mathbf{1}_K^T\right| \\ &= \left(\frac{L+1}{L}\right)^K \left(\frac{L-K+1}{L+1}\right)\end{aligned} \tag{18}$$

and this reaches the lower bound in Proposition 5. If there exists a quaternary Hadamard matrix $\mathbf{H}_Q$ of size $L-1$, by (16) we can obtain a minimum-TSC quaternary set with TAE

$$\left|\mathcal{S}_Q^H \mathcal{S}_Q\right| = \left(\frac{L-1}{L}\right)^K \left(\frac{L+K-1}{L-1}\right) \qquad (19)$$

and this is the upper bound value in Proposition 5. ■

PROOF OF PROPOSITION 6

Part A

(*i*) If $L \equiv 0 (\text{mod } 2)$ and $\mathcal{S}_Q$ achieves the TSC lower bound in Table I, it has orthogonal columns, i.e. $\mathcal{S}_Q^H \mathcal{S}_Q = \mathbf{I}_K$. Therefore,

$$\begin{aligned} C_{\text{sum}}(\mathcal{S}_Q) &= \log_2 \left|\mathbf{I}_K + \gamma \mathcal{S}_Q^H \mathcal{S}_Q\right| \\ &= \log_2 \left|(1+\gamma)\mathbf{I}_K\right| \\ &= K \log_2(1+\gamma). \end{aligned} \qquad (20)$$

(*ii*) By Proposition 4, the minimum-TSC quaternary set $\mathcal{S}_Q$ has following properties: 1) $[\mathbf{I}_K + \gamma \mathcal{S}_Q^H \mathcal{S}_Q]_{mm} = 1+\gamma$, $m = 1, 2, \ldots, K$; 2) $\left|[\mathbf{I}_K + \gamma \mathcal{S}_Q^H \mathcal{S}_Q]_{mn}\right| = \frac{\gamma}{L}$, $m \neq n$, $m = 1, 2, \ldots, K$, $n = 1, 2, \ldots, K$. Then, *Lemma 2* of [16] implies that the determinant of $\mathbf{I}_K + \gamma \mathcal{S}_Q^H \mathcal{S}_Q$ is bounded as follows:

$$\begin{aligned} &(1+\gamma+\tfrac{\gamma}{L})^{(K-1)}(1+\gamma-(K-1)\tfrac{\gamma}{L}) \\ &\quad \leq \left|\mathbf{I}_K + \gamma \mathcal{S}_Q^H \mathcal{S}_Q\right| \\ &\leq (1+\gamma-\tfrac{\gamma}{L})^{(K-1)}(1+\gamma+(K-1)\tfrac{\gamma}{L}). \end{aligned} \qquad (21)$$

Therefore, $C_{\text{sum}}(\mathcal{S}_Q) = \log_2 \left|\mathbf{I}_K + \gamma \mathcal{S}_Q^H \mathcal{S}_Q\right|$ is bounded as

$$\begin{aligned} &(K-1)\log_2(1+\tfrac{L+1}{L}\gamma) + \log_2(1+\tfrac{L-K+1}{L}\gamma) \\ &\quad \leq C_{\text{sum}}(\mathcal{S}_Q) \\ &\leq (K-1)\log_2(1+\tfrac{L-1}{L}\gamma) + \log_2(1+\tfrac{L+K-1}{L}\gamma). \end{aligned} \qquad (22)$$

If there exists a quaternary Hadamard matrix $\mathbf{H}_Q$ of size $L+1$, by Proposition 4, part (*ii*), we can obtain a minimum-TSC quaternary set that satisfies (16). Therefore,

$$\begin{aligned} C_{\text{sum}}(\mathcal{S}_Q) &= \log_2 \left|\mathbf{I}_K + \gamma \mathcal{S}_Q^H \mathcal{S}_Q\right| \\ &= \log_2 \left|\left(1+\frac{L+1}{L}\right)\mathbf{I}_K - \frac{\gamma}{L}\mathbf{1}_K \mathbf{1}_K^T\right| \\ &= (K-1)\log_2\left(1+\frac{L+1}{L}\gamma\right) \\ &\quad + \log_2\left(1+\frac{L-K+1}{L}\gamma\right) \end{aligned} \qquad (23)$$

which is equal to the lower bound in Proposition 6, *Part A(ii)*.

If there exists a quaternary Hadamard matrix $\mathbf{H}_Q$ of size $L-1$ and $K \leq L-1$, we can obtain a minimum-TSC quaternary set that satisfies (16) and by similar to (23) derivation we can evaluate C_{sum} as follows:

$$\begin{aligned} C_{\text{sum}}(\mathcal{S}_Q) &= \log_2 \left|\mathbf{I}_K + \gamma \mathcal{S}_Q^H \mathcal{S}_Q\right| \\ &= (K-1)\log_2\left(1+\frac{L-1}{L}\gamma\right) \\ &\quad + \log_2\left(1+\frac{L+K-1}{L}\gamma\right) \end{aligned} \qquad (24)$$

which is the upper bound in Proposition 6, *Part A(ii)*.

Part B

Set $\mathbf{D} \triangleq \sqrt{\frac{L}{K}}\mathcal{S}_Q^H$ Then

$$\begin{aligned} C_{\text{sum}}(\mathcal{S}_Q) &= \log_2 \left|\mathbf{I}_L + \gamma \mathcal{S}_Q \mathcal{S}_Q^H\right| \\ &= \log_2 \left|\mathbf{I}_L + \gamma \frac{K}{L}\mathbf{D}^H \mathbf{D}\right| \\ &= \log_2 \left|\mathbf{I}_K + \gamma \frac{K}{L}\mathbf{D}\mathbf{D}^H\right|. \end{aligned} \qquad (25)$$

$\mathbf{D} \in \frac{1}{\sqrt{K}}\{\pm 1, \pm j\}^{K \times L}$ can be viewed as a signature matrix with L unit-norm quaternary signatures of length $K \geq L$. Therefore, $C_{\text{sum}}(\mathcal{S}_Q)$ at SNR γ equals $C_{\text{sum}}(\mathbf{D})$ at SNR $\gamma\frac{K}{L}$ where $\mathcal{S}_Q$ is the overloaded and $\mathbf{D}$ is the corresponding underloaded set. We can show that if TSC($\mathcal{S}_Q$) achieves the TSC lower bound for overloaded sets in Table II, then TSC($\mathbf{D}$) achieves the TSC lower bound for underloaded sets in Table I. Hence, we can apply our results in *Part A* of Proposition 6 to $\mathbf{D}$ and obtain the $C_{\text{sum}}(\mathcal{S}_Q)$ expressions in all cases of Proposition 6, *Part B*, directly. ■

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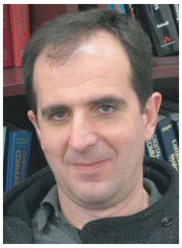

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